EE327 Digital Signal Processing DFT Properties

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REMIDER 1. Applications of DFT

- Finding Signal's Frequency Spectrum
  - Understanding frequency contents of signals
- Finding System's Frequency Response
  - Analyzing Systems in the Frequency domain
- Intermediate Step for other operations
  - FFT convolution

REMINDER 2. Information Coding in Signals

- Information in the time domain
  - Shape
  - Examples:
    - Readings of a sensor over time
    - Stock market signals

- Information in frequency domain
  - Amplitude
  - Phase
  - Frequency
  - Examples:
    - FM radio information
    - 50Hz noise

REMINDER 3. Understanding Signal's Frequency Spectrum

1. Collect data
2. Use DFT to convert it to frequency domain
3. Convert it to polar coordinates
4. If needed do this many times and average the results

5. Now study the spectrum

REMINDER 4: System's Frequency Response

- Frequency response of a system $H[f]$:
  - Complete description of how it changes the amplitude and phase of input sinusoidals in the output.

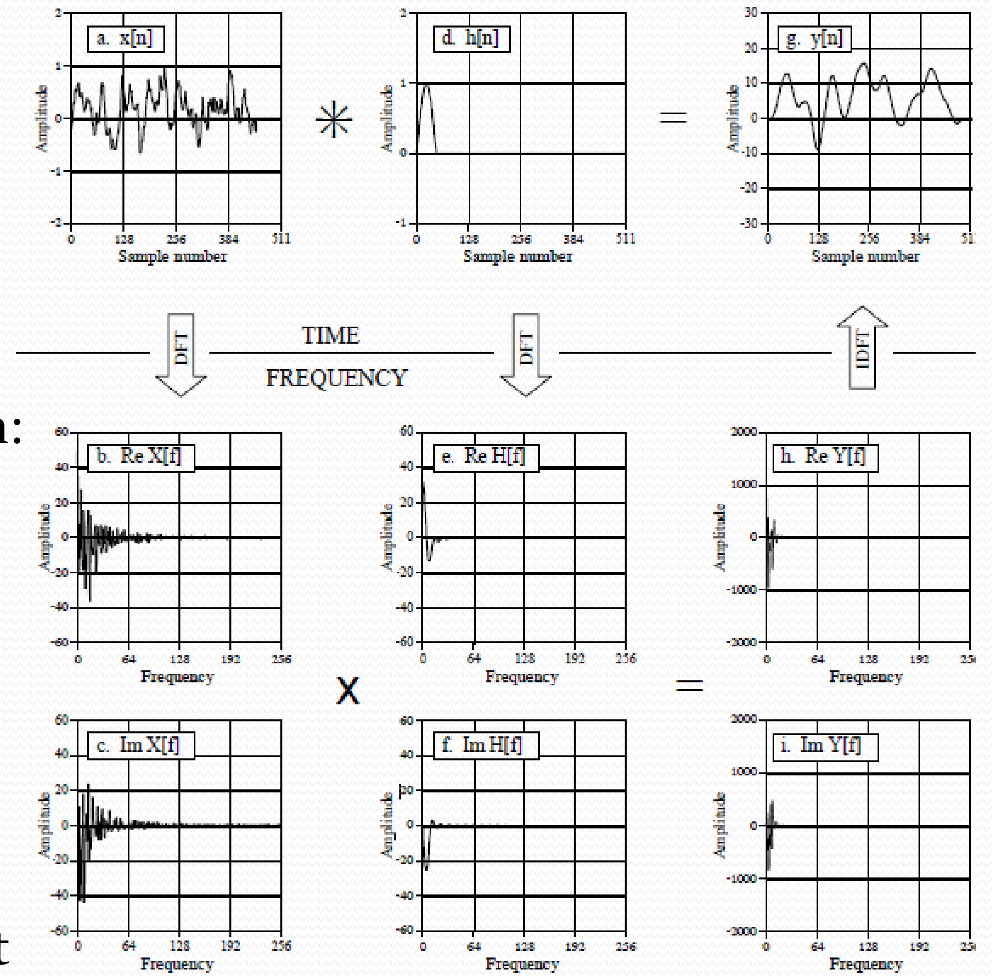
- System's frequency response = Fourier Transform of its impulse response

- $h[t] \xrightarrow{\text{FT}} H[f]$

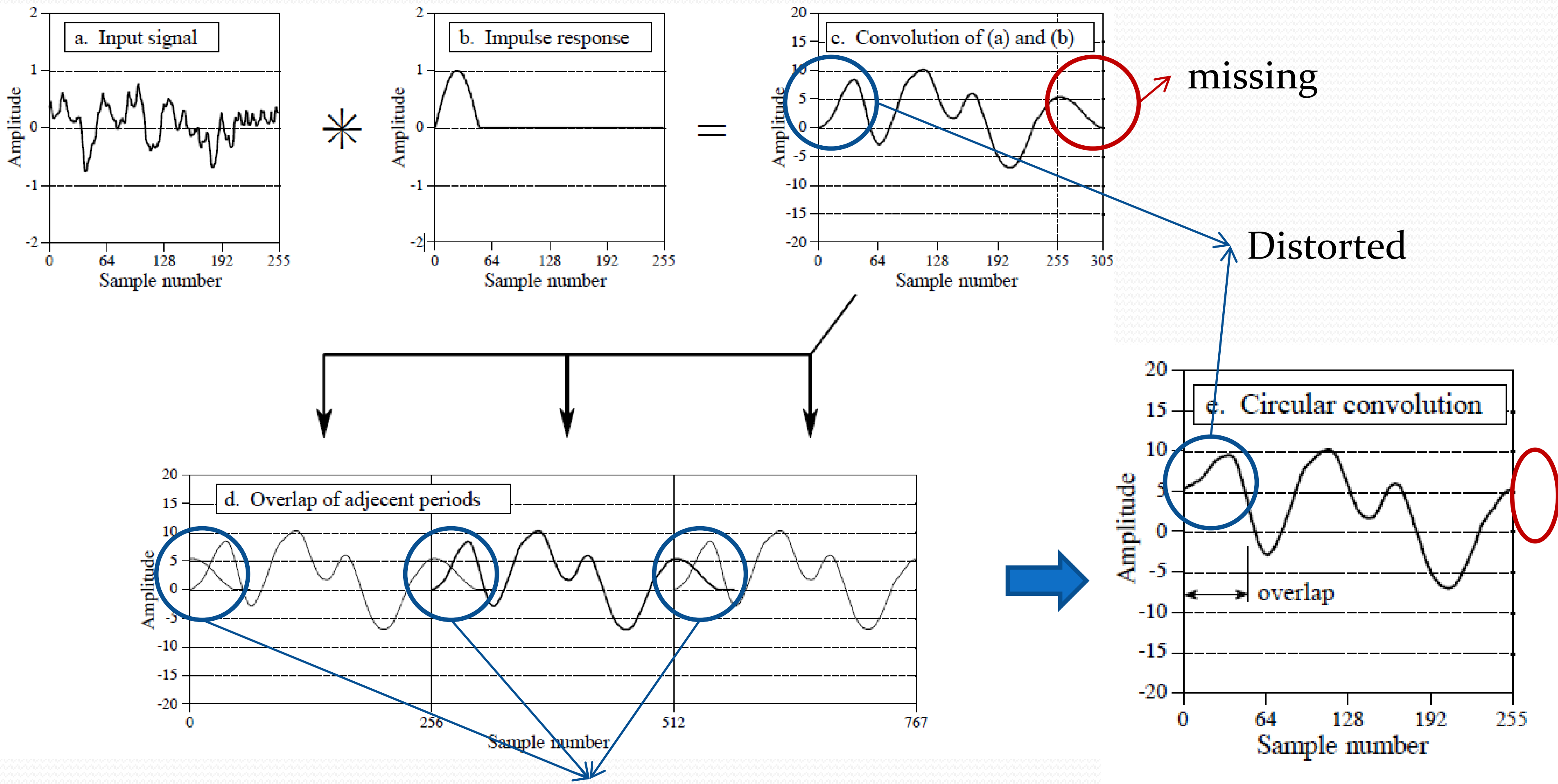
REMINDER 5: Convolution via Frequency domain

- c[n]= a[n]*b[n]

1. Pad both signals to N+M-1 points by adding zeros
2. Convert both to frequency domain:
   - MagA[f], Mag[f]
   - PhaseB[f],Phase[f]
3. Multiply in frequency domain:
   - MagC[f]=MagA[f]×MagB[f]
   - PhaseC[f]=PhaseA[f]+PhaseB[f]
4. Convert C[f] to time domain to get c[n] EXACTLY



REMINER 6: Circular Convolution Problem



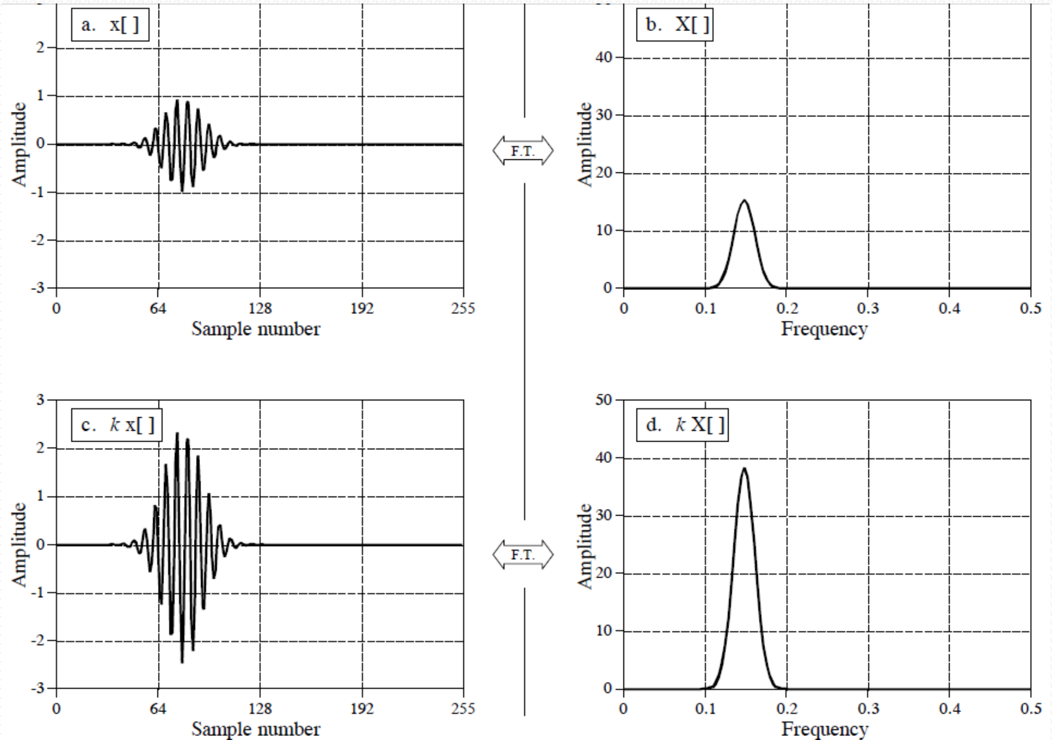
Cause of Distortion: DFT assumes the signal is periodic

Properties of Fourier Transform

- We have two domains:
  - Time Domain
  - Frequency Domain
- How does a mathematical change in one domain affect the signal in the other domain?

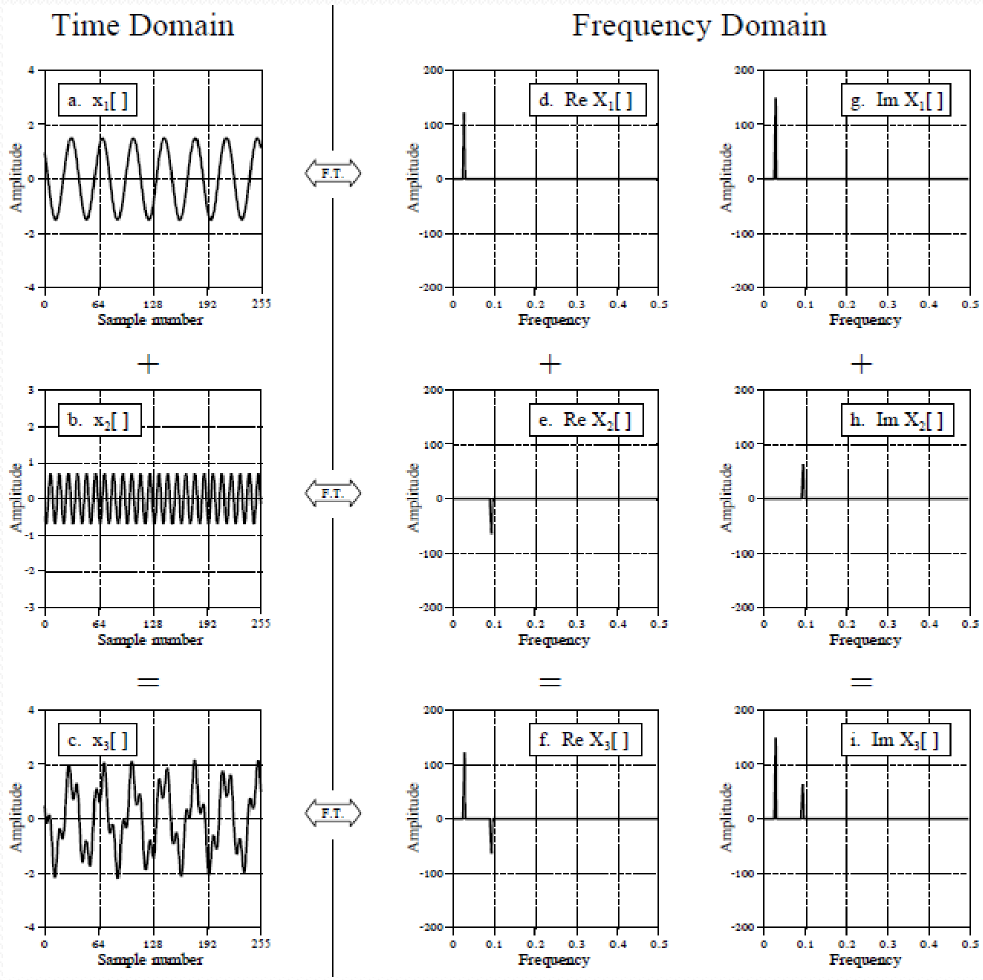
Fourier Transform is Additive

- Scaling of the amplitude in one domain produces a scaling with the SAME factor in the amplitude of the other domain.
- *Use polar or rectangular coordinates*



Fourier Transform is Homogeneous

- Addition in one domain results in addition in the other domain.
- *MUST Use rectangular coordinates*
- *NEVER add magnitudes in the frequency domain*



FT is Linear

- Additive+Homogeneous = Linear

REMINDER: Linear Phase

Time Domain	Frequency Domain
Symmetric around ITS CENTER	Zero phase
Linear around something else	Linear phase
Asymmetric	Nonlinear phase

Time Shift = Phase offset

$$x[n] \xleftrightarrow[IDFT]{DFT} MagX[f], PhaseX[f]$$

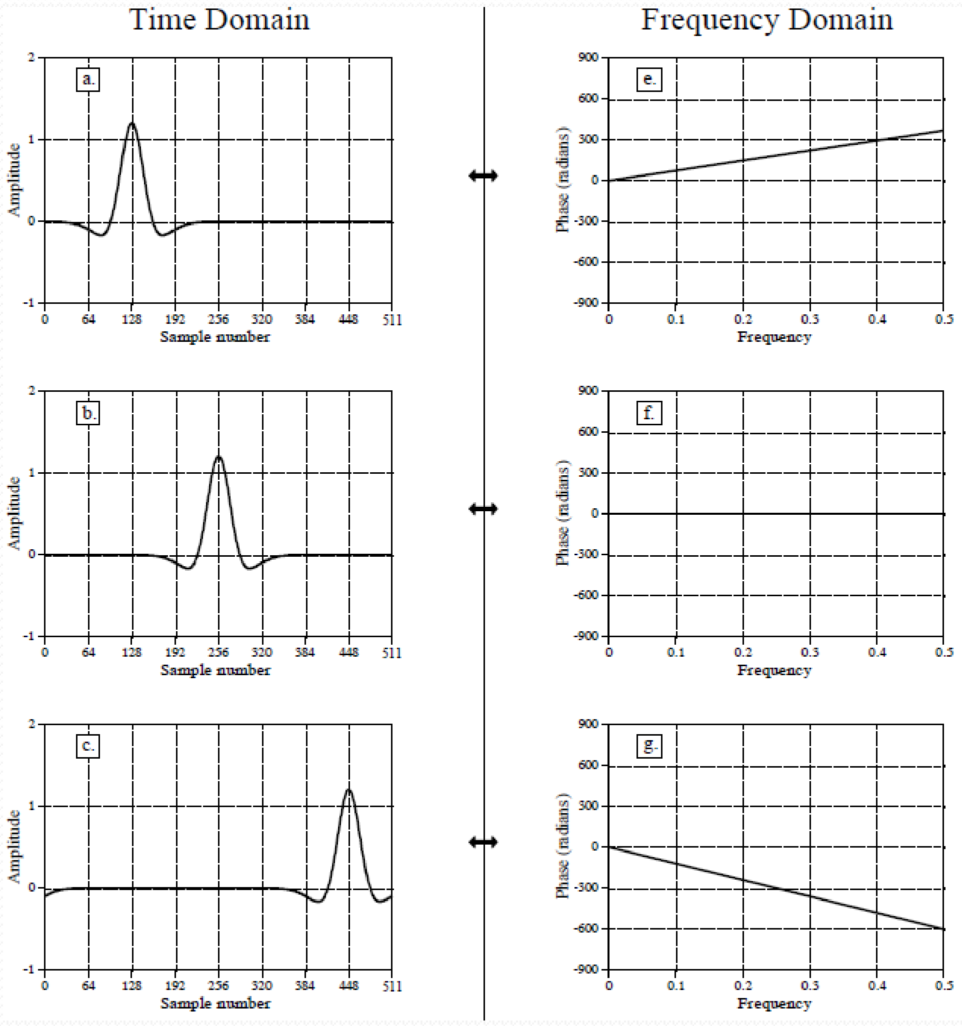
$$\Downarrow$$

$$x[n+s] \xleftrightarrow[IDFT]{DFT} MagX[f], PhaseX[f] + 2\pi sf$$

- Right Shift in time domain ←→Decrease in slope of the phase

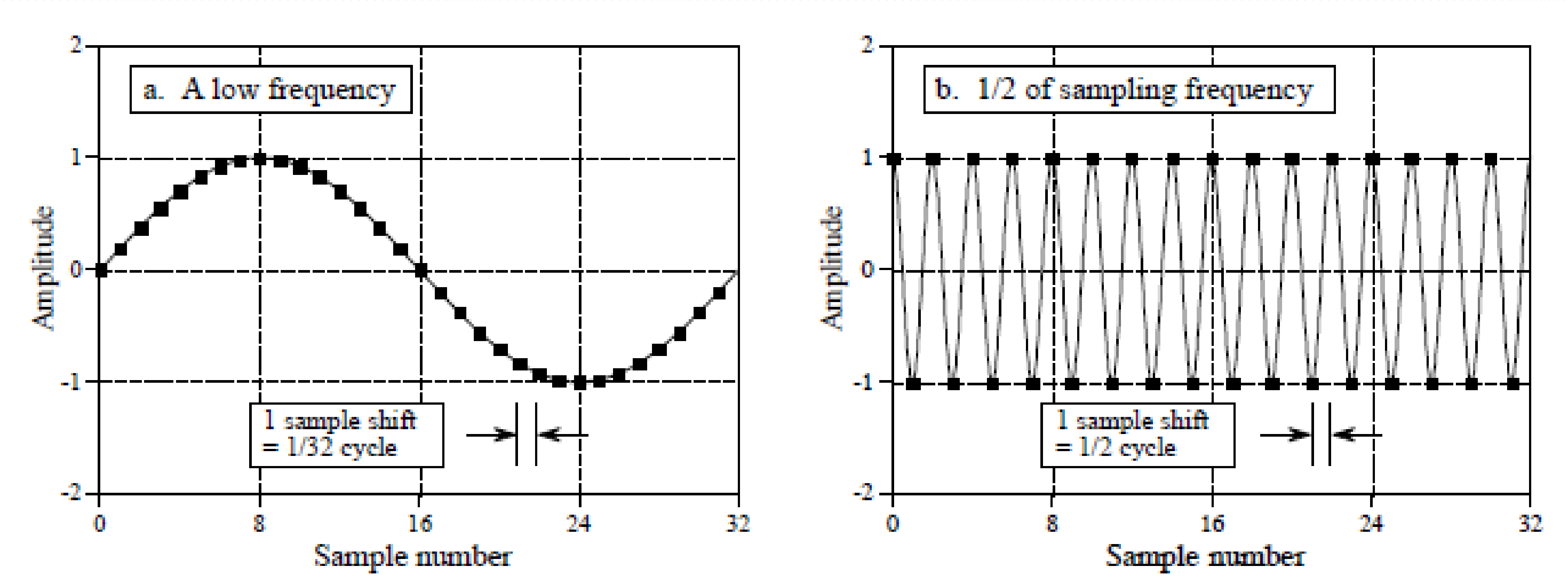
Phase will be drawn unwrapped in this lecture

Example Time Shift

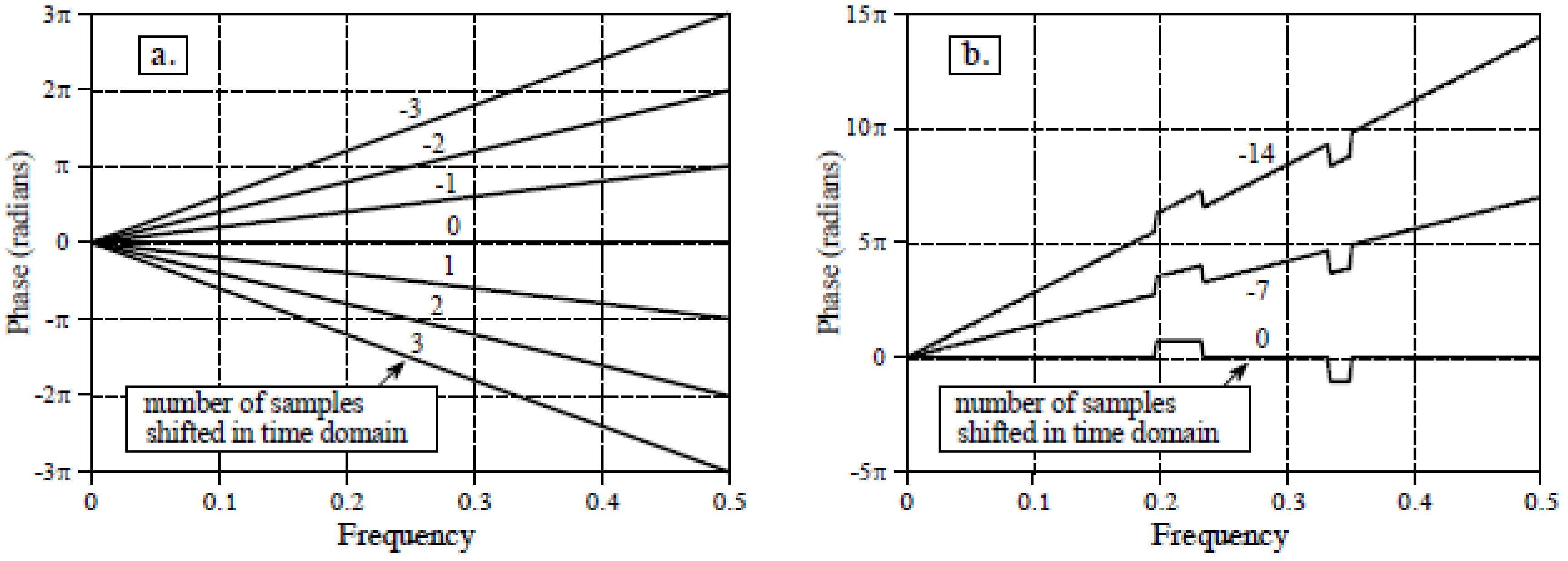


WHY?

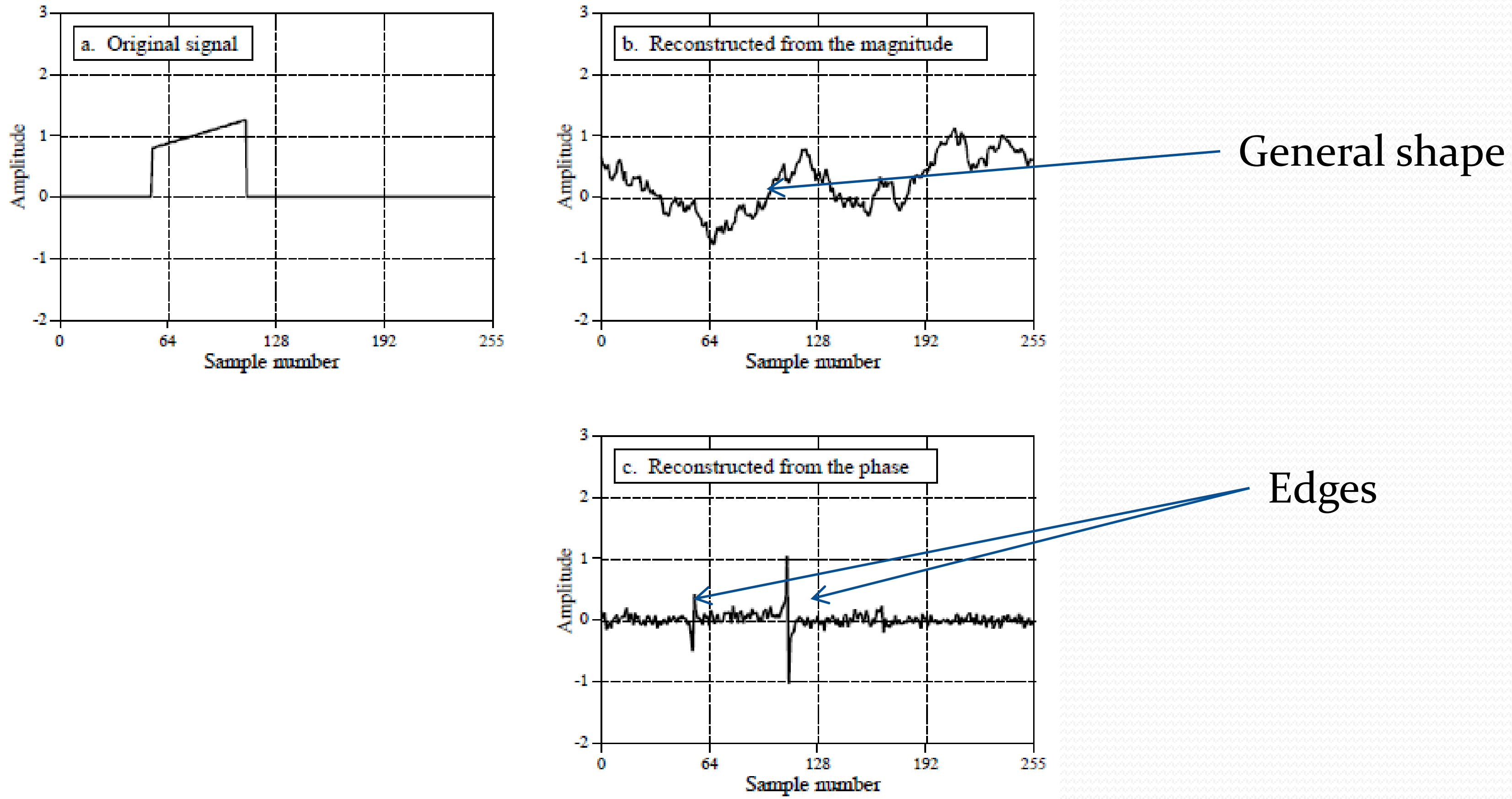
- 1 sample shift in time domain ←→shift all sinusoidals
- The frequency response in this case is PROPORTIONAL to the frequency



Effect of time shift in general

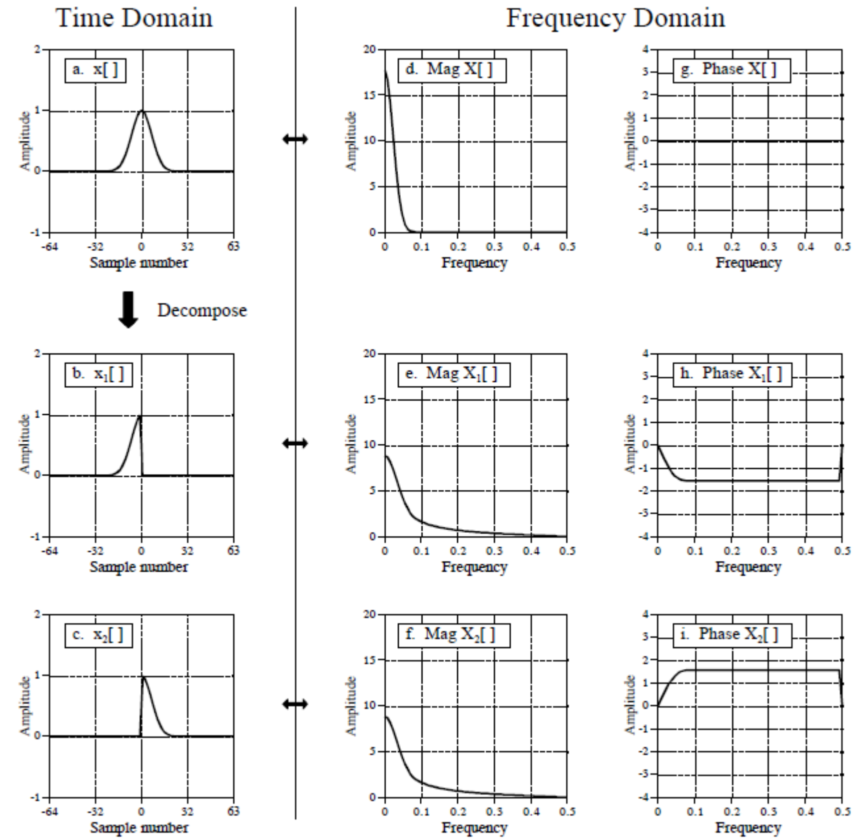


What is encoded in the phase



Why symmetry result in linearity

- Why does symmetric signals appear as linear phase?



Time Flipping

- Flipping time domain negates the phase

$$x[n] \xleftrightarrow[IDFT]{DFT} \operatorname{Re} X[f], \operatorname{Im} X[f]$$

$$x[-n] \xleftrightarrow[IDFT]{DFT} \operatorname{Re} X[f], -\operatorname{Im} X[f]$$

Complex conjugate

- Change the sign of the imaginary part
- Complex conjugate of A[*f*] is called A*[*f*]

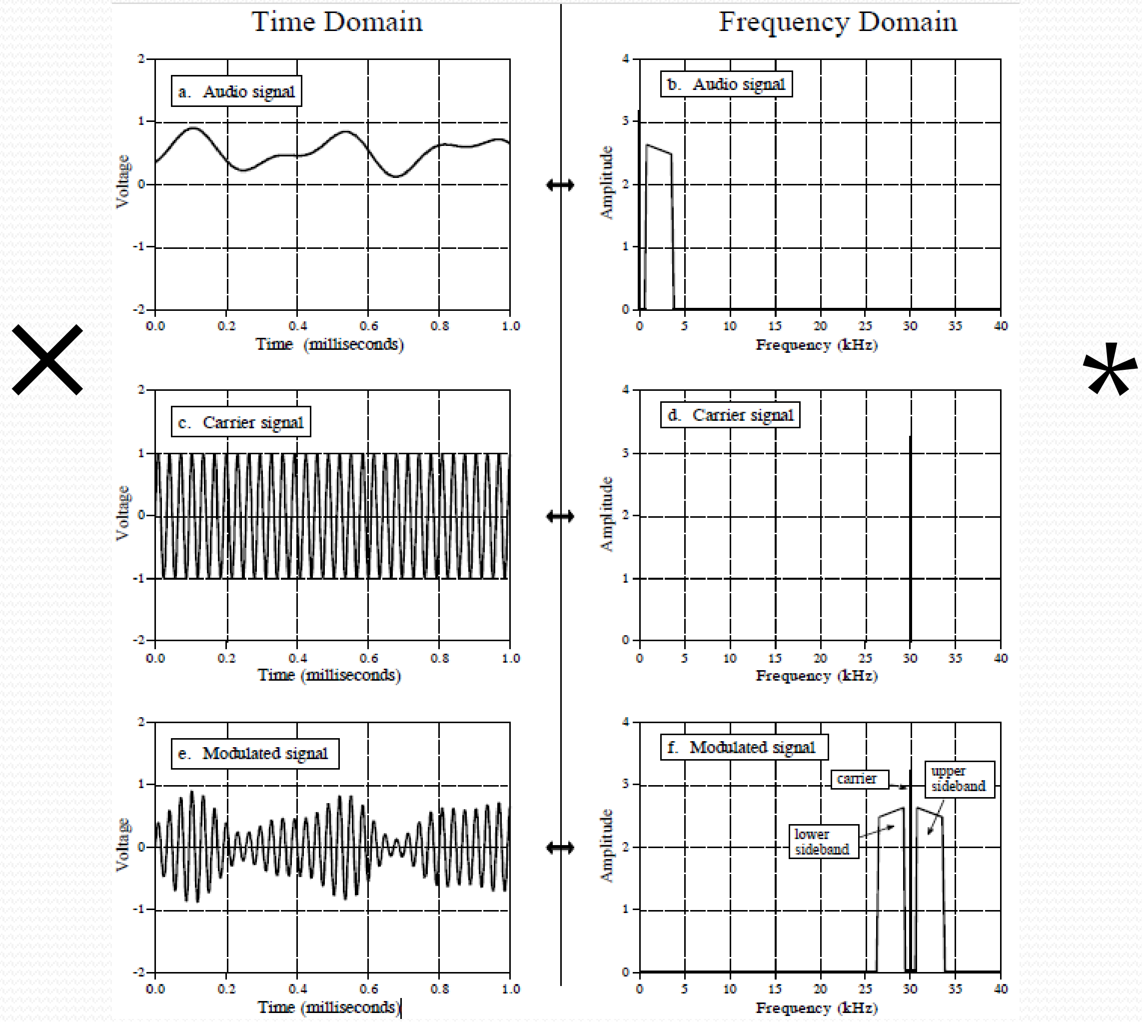
Applications of Time flipping

- Convolution = $a[n]*b[n]$
- Correlation= $a[n]*b[-n]$
- $a[n]*b[n] \xleftrightarrow[IDFT]{DFT} A[f]\times B[f]$
- $a[n]*b[-n] \xleftrightarrow[IDFT]{DFT} A[f]\times B*[f]$

Compression, Expansion and Multirate methods

- SELF READ

Amplitude Modulation



DTFT

- Decomposition

$$ReX(\omega) = \sum_{n=-\infty}^{+\infty} x[n]\cos(\omega n)$$

$$ImX(\omega) = -\sum_{n=-\infty}^{+\infty} x[n]\sin(\omega n)$$

- Synthesis

$$x[n] = \frac{1}{\pi}\int_{0}^{\pi} ReX(\omega)\cos(\omega n) - ImX(\omega)\sin(\omega n)\,d\omega$$

DFT and DTFT

- DFT is roughly the sampling of DTFT
- DFT is used in computer programs
- DTFT is used in mathematical computations

Parserval's Relation (Not in Exam)

$$\sum_{i=0}^{N-1} x[i]^2 = \frac{2}{N} \sum_{k=0}^{N/2} Mag\,X[k]^2$$