EE327 Digital Signal Processing Statistics and Noise

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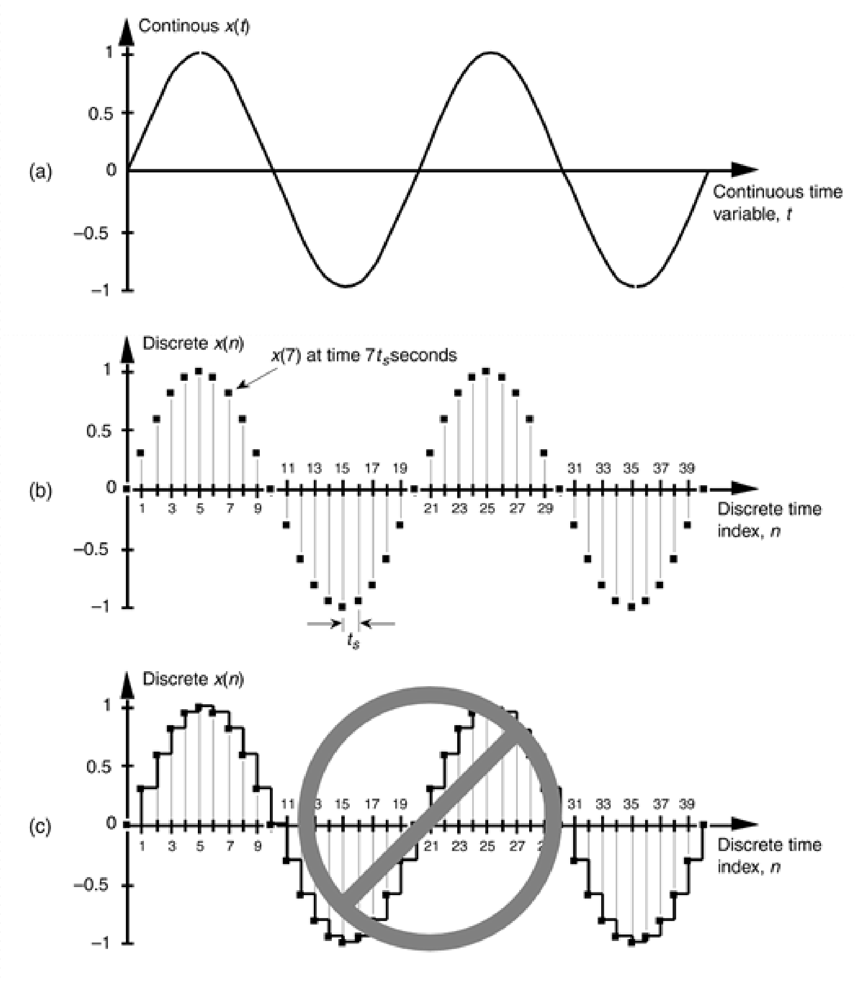
REMINDER 1
What is a discrete signal



Continuous Signal $x(t) = \sin(2\pi f_o t)$

Discrete Signal $x[n] = \sin(2\pi f_0 n T_s)$

????? Signal

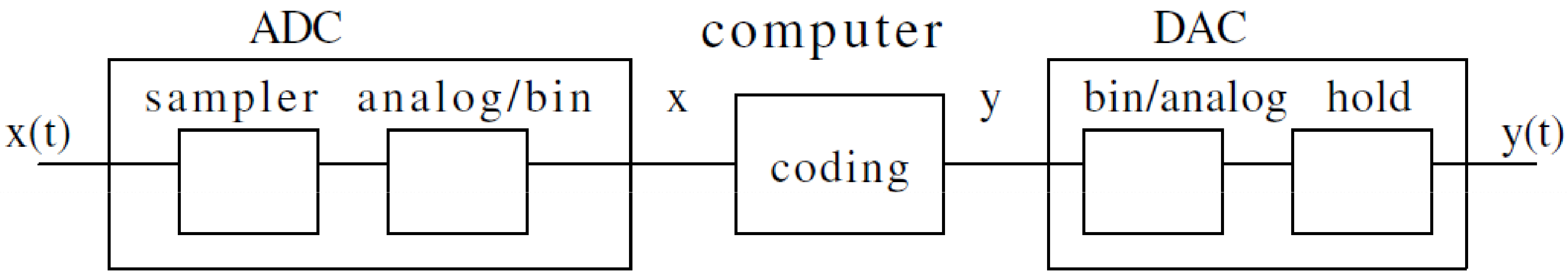
REMINDER 2
Types of Signals

		Independent	
Dependent		Discrete	Continuous
	Discrete	Quantized Discrete Signal (Digital Signal) $x[n]$	Quantized Signal $\bar{x}(t)$
	Continuous	Discrete or Digitized Signal $\tilde{x}[n]$	Continuous or Analog Signal $x(t)$

DAC

ADC

REMINDER 3
General DSP System



$$y = (b_{-1}y_{-1} + \cdots + b_{-m}y_{-m}) + (ax + a_{-1}x_{-1} + \cdots + a_{-n}x_{-n})$$

$$y = \sum_{i=1}^{m} b_{-i}y_{-1} + \sum_{j=0}^{n} a_{-j}x_{-1}$$

Nearly ALL DSP that you need is contained in this equation

Mean and Standard Deviation

- Mean

$$\mu = \frac{1}{N}\sum_{i=0}^{N-1} x[i] = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

$$\mu = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)dt$$

- Variance

$$\sigma^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i - \mu)^2$$

$$\sigma^2 = \lim_{T\to\infty}\frac{1}{T}\int_0^T (x(t)-\mu)^2 dt$$

- Root Mean Square (quadratic mean)

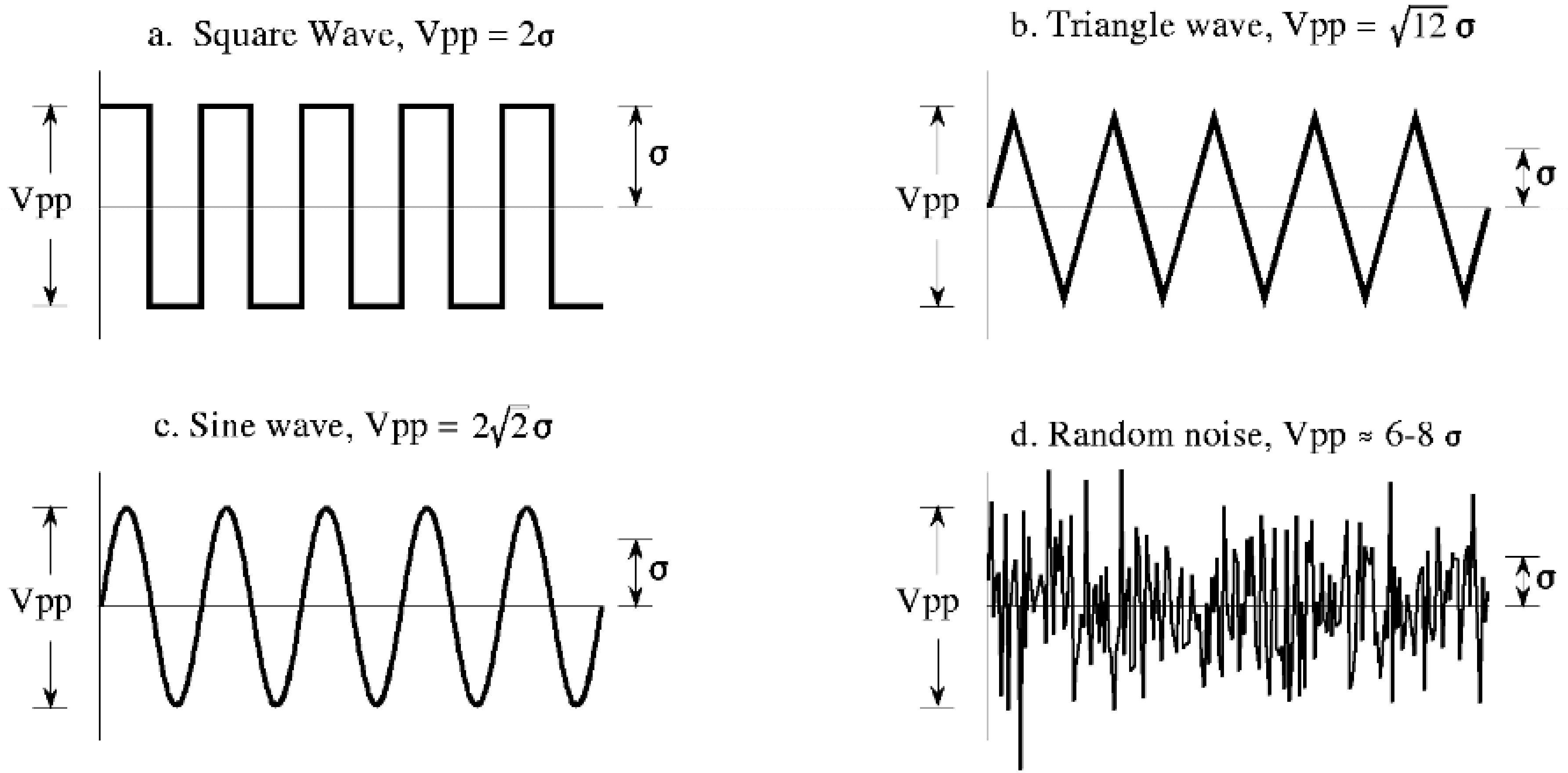
$$RMS = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1}(x_i)^2}$$

$$RMS = \lim_{T\to\infty}\sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$$

All continuous formulas assume that x(t) is zero for t<o or x(-t)=x(t)

Self test: Find the relation between RMS and variance for signals with zero mean

Common variance formulas



Calculating the Mean and Variance

```
100 CALCULATION OF THE MEAN AND STANDARD DEVIATION
110 '
120 DIM X[511]                          'The signal is held in X[0] to X[511]
130 N% = 512                            'N% is the number of points in the signal
140 '
150 GOSUB XXXX                          'Mythical subroutine that loads the signal into X[ ]
160 '
170 MEAN = 0                            'Find the mean via Eq. 2-1
180 FOR I% = 0 TO N%-1
190   MEAN = MEAN + X[I%]
200 NEXT I%
210 MEAN = MEAN/N%
220 '
230 VARIANCE = 0                        'Find the standard deviation via Eq. 2-2
240 FOR I% = 0 TO N%-1
250   VARIANCE = VARIANCE + ( X[I%] - MEAN )^2
260 NEXT I%
270 VARIANCE = VARIANCE/(N%-1)
280 SD = SQR(VARIANCE)
290 '
300 PRINT  MEAN  SD                     'Print the calculated mean and standard deviation
310 '
320 END
```

Calculating the Mean and Variance Better way (Running Statistics)

$$\sigma^2 = \frac{1}{N-1}\left[\sum_{i=0}^{N-1} x_i^2 - \frac{1}{N}\left(\sum_{i=0}^{N-1} x_i\right)^2\right]$$

```
100 'MEAN AND STANDARD DEVIATION USING RUNNING STATISTICS
110 '
120 DIM X[511]                  'The signal is held in X[0] to X[511]
130 '
140 GOSUB XXXX                  'Mythical subroutine that loads the signal into X[ ]
150 '
160 N% = 0                      'Zero the three running parameters
170 SUM = 0
180 SUMSQUARES = 0
190 '
200 FOR I% = 0 TO 511           'Loop through each sample in the signal
210   '
220   N% = N%+1                 'Update the three parameters
230   SUM = SUM + X[I%]
240   SUMSQUARES = SUMSQUARES + X[I%]^2
250   '
260   MEAN = SUM/N%             'Calculate mean and standard deviation via Eq. 2-3
270   IF N% = 1 THEN SD = 0: GOTO 300
280   SD = SQR( (SUMSQUARES - SUM^2/N%) / (N%-1) )
290   '
300   PRINT  MEAN   SD          'Print the running mean and standard deviation
310   '
320 NEXT I%
330 '
340 END
```

Signal to noise ratio

$$SNR = \frac{\text{power of signal}}{\text{power of noise}}$$

Coefficient of variance

$$CV = \frac{\text{standard deviation}}{\text{mean}}$$

SNR and CV

- **Sometimes:**

signal = mean, noise=variance

In these cases

$$SNR = \frac{\mu}{\sigma} \qquad CV = \frac{\sigma}{\mu} \times 100\%$$

We want high SNR and low CV

Probability and Statistics

- Statistics describe acquired signal
- Probabilities describes underlying process

- Difference is the statistical fluctuations

- Typical error is

$$\frac{\sigma}{N^{\frac{1}{2}}}$$

Variance equation again

- Variance of acquired signal:

$$\sigma^2 = \frac{1}{N}\sum_{i=0}^{N-1}(x_i - \mu)^2$$

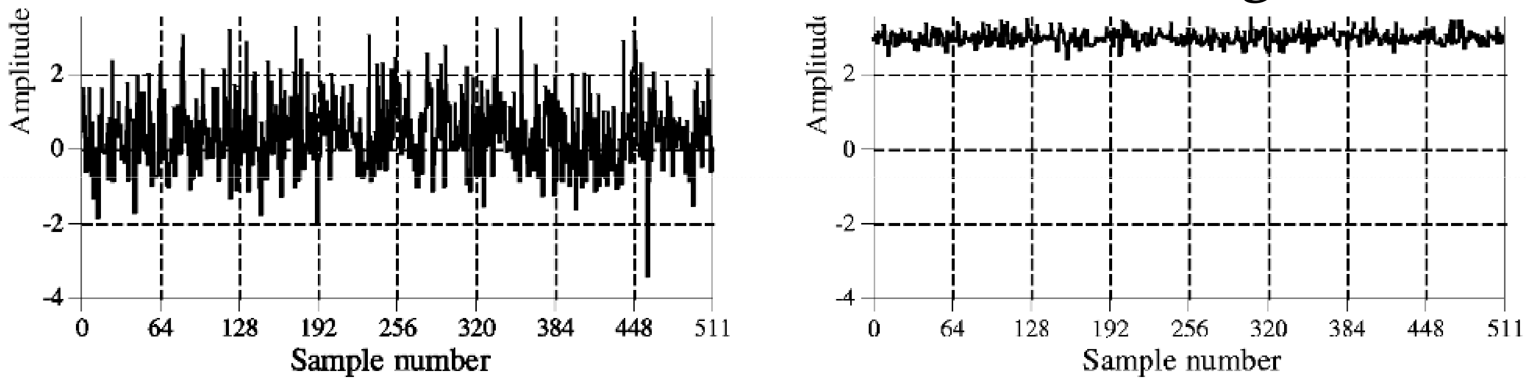
- ESTIMATED Variance of underlying process

$$\sigma^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i - \mu)^2$$

Examples of mean and variance

FAST TEST: These signals are discrete or continuous?
What is the domain of these signals?

Stationary



nonstationary