EE327 Digital Signal Processing Linear Systems and Signals

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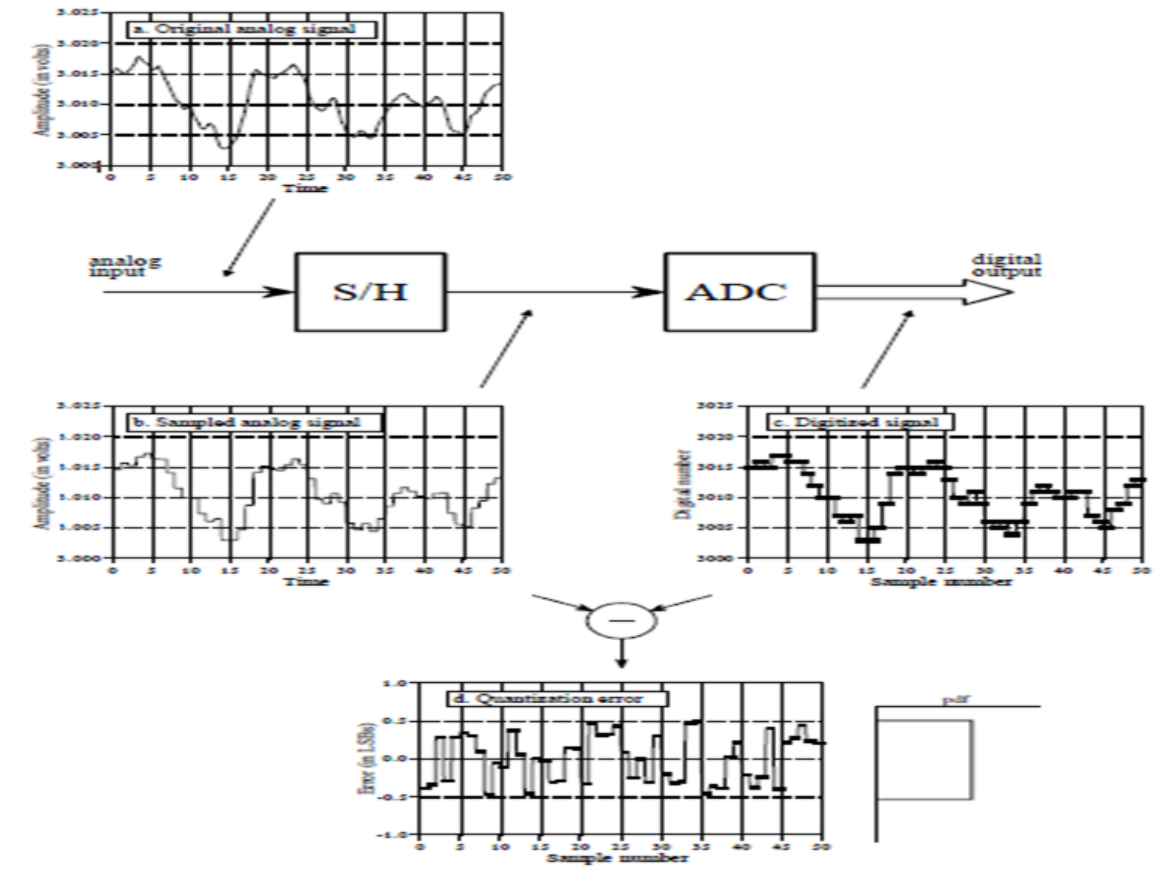
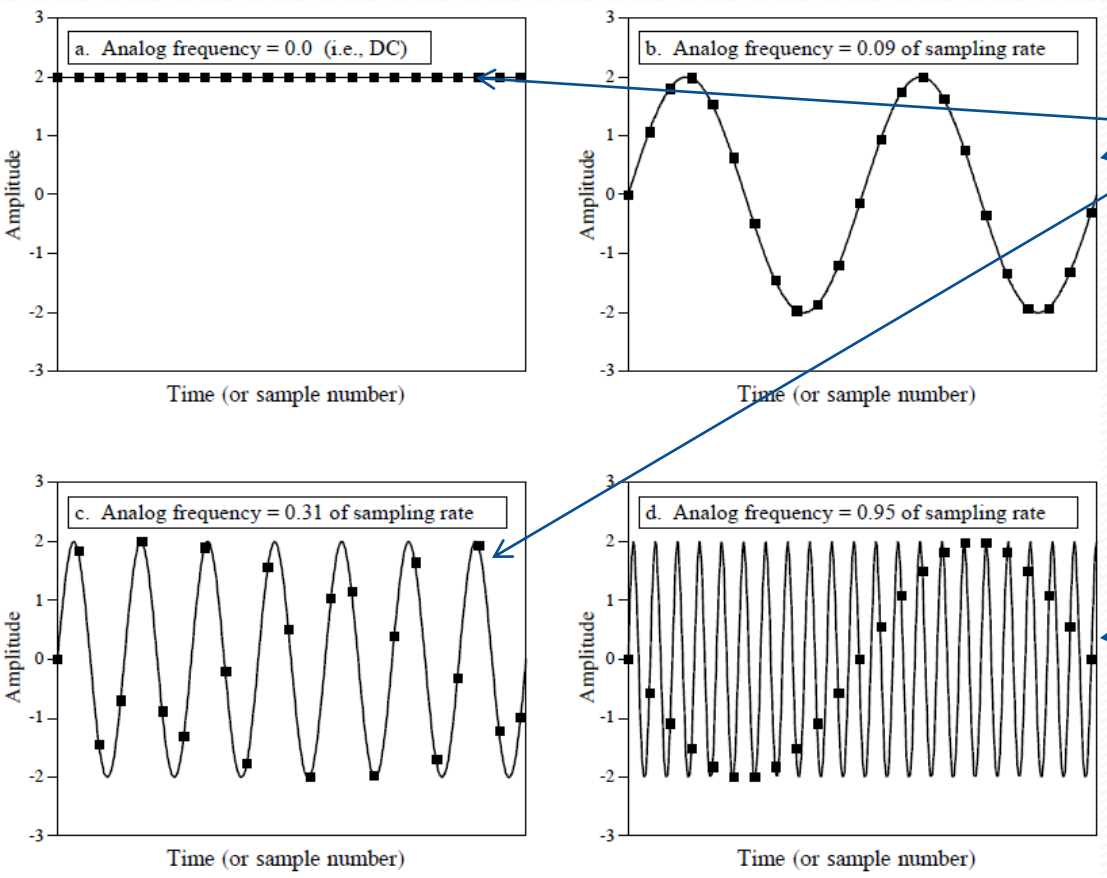
REMINDER 1: ADC



REMINDER 2: Sampling

- Our goal is to be able to reconstruct the analog signals completely from the digitized version (ignoring quantization).



REMINDER 3: Nyquist Frequency

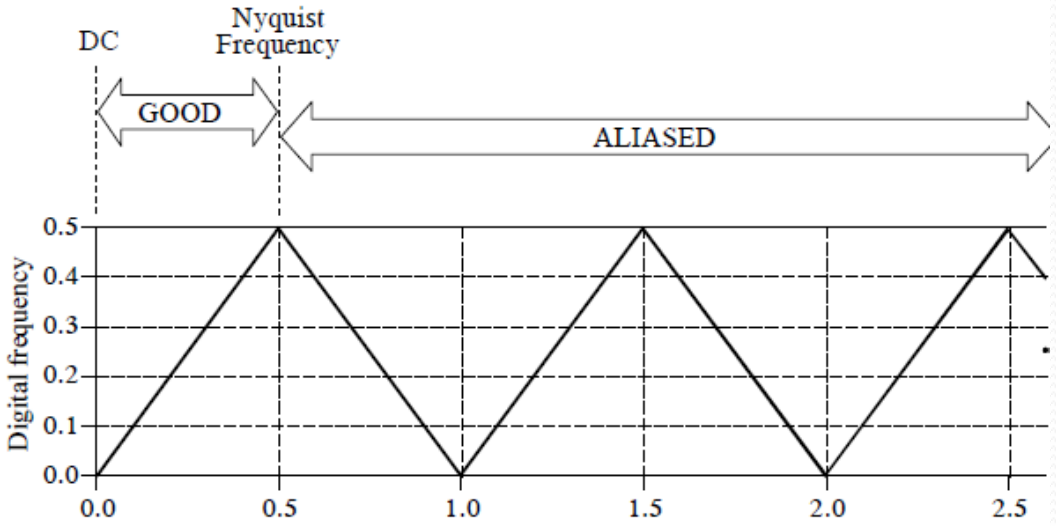
- Half the sampling rate

- The maximum frequency representable in the discrete signal without aliasing

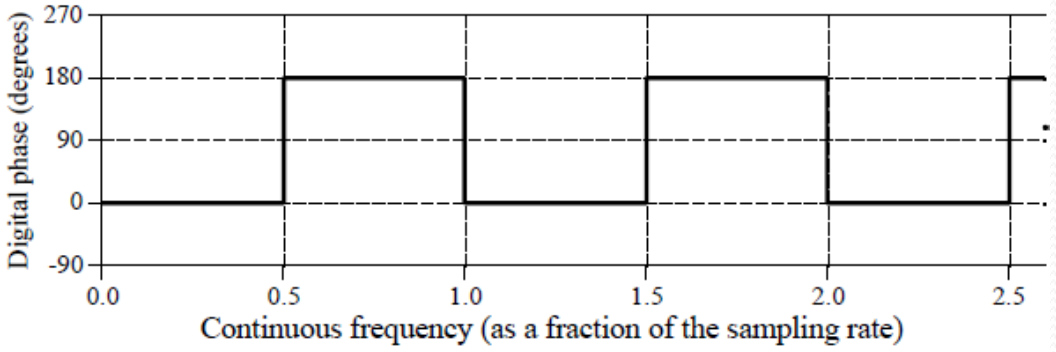
$$f_n = \frac{f_s}{2}$$

REMINDER 4: Aliasing

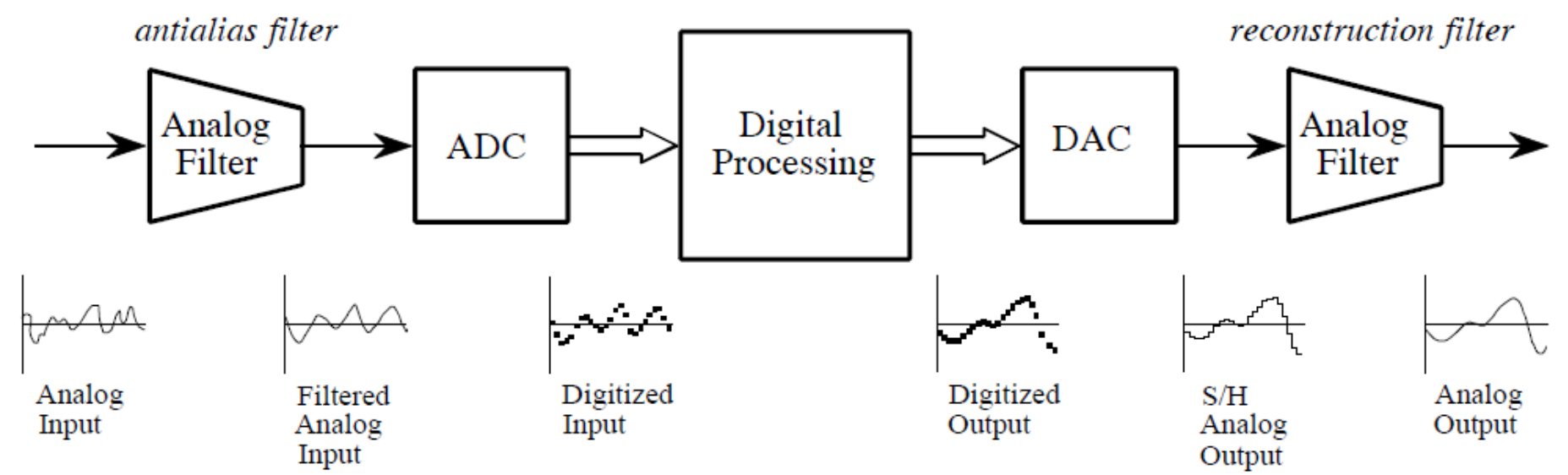
Aliasing causes information loss about both high and low frequencies



Aliasing causes a phase shift of π or zero as follows



REMINDER 5: Complete ADC/DAC system



SELF TEST: Why do we need an antialiasing filter even if we are not interested in signals over the Nyquest frequency?

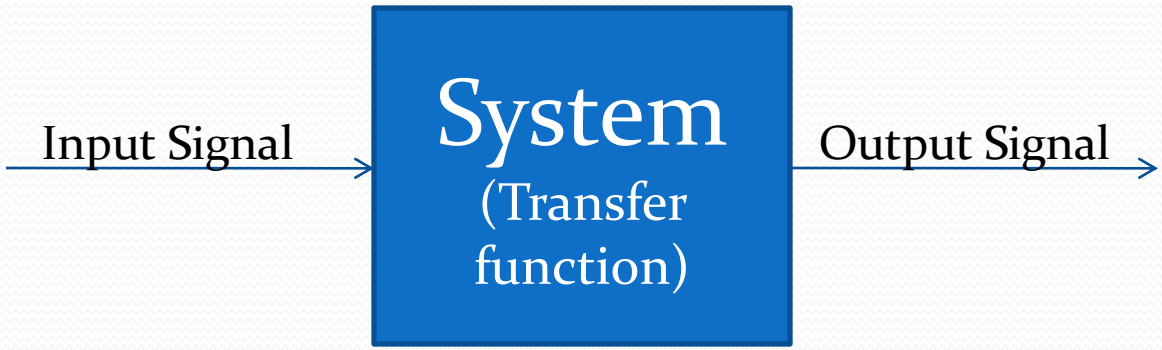
Let is play a game

- What is in the box
  - Elephant
  - Linear System
  - Nonlinear System

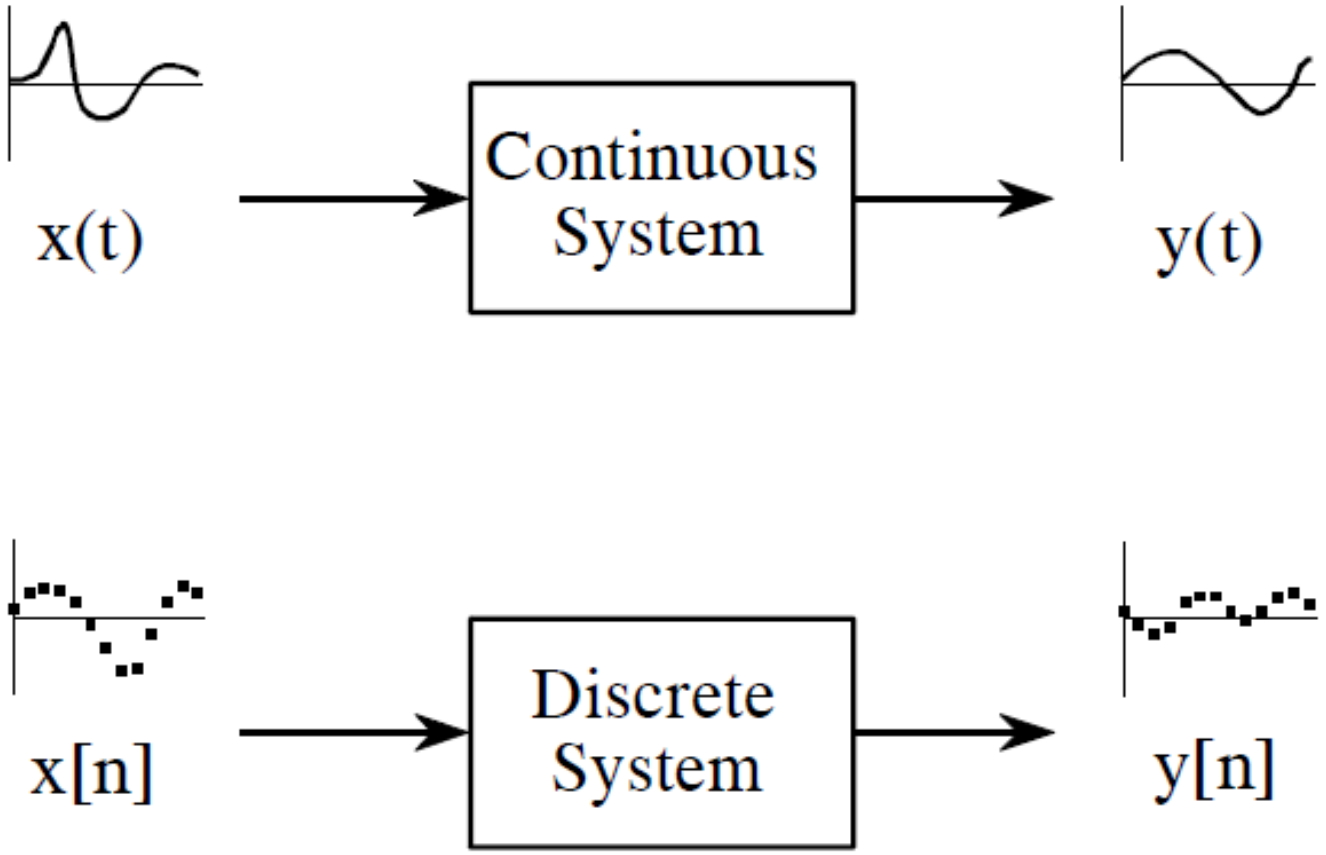
- Ask me

Signal and System

- Signal
  - Description of how a quantity(s) is varying with some parameter(s)
- System
  - Any process that produces an output signal in response to an input signal

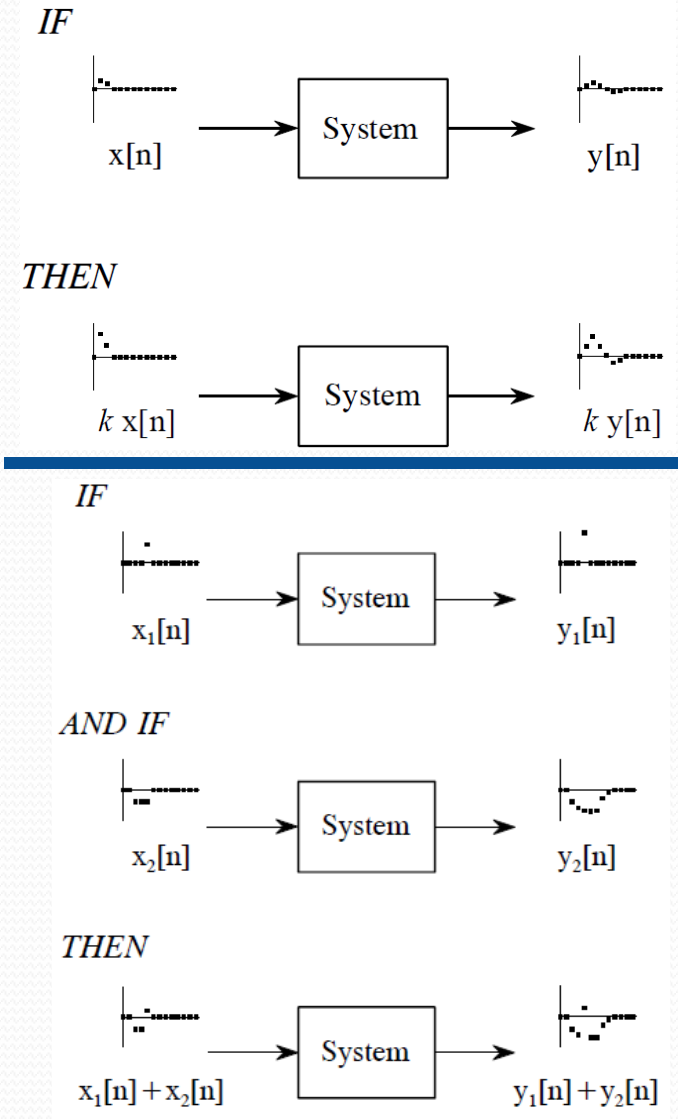


Types of Systems



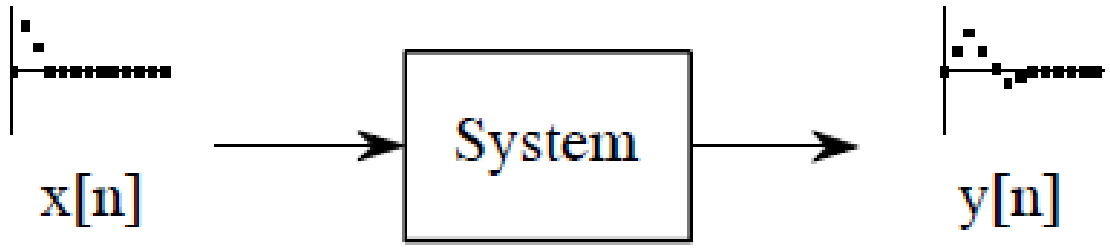
Linear Systems

- Linear = Homogeneous+Additive
- Homogeneity
  - If X[n]→Y[n]
    then k X[n] → k Y[n]
- Additive
  - If X1[n] → Y1[n] and X2[n]→Y2[n]
    then X1[n]+X2[n] → Y1[n]+Y2[n]
- Most DSP linear systems are also *shift invariant* (LTI)

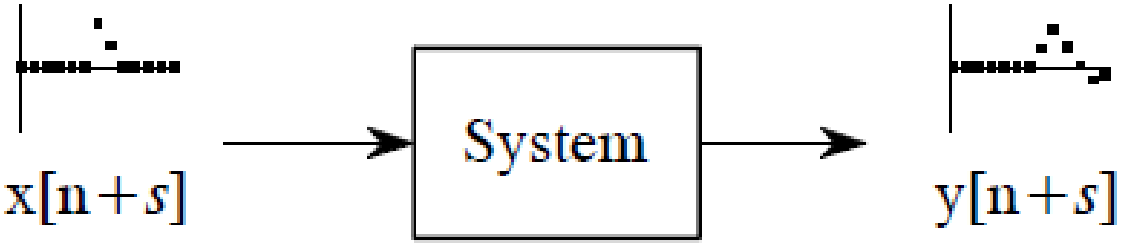


Shift Invariance

IF



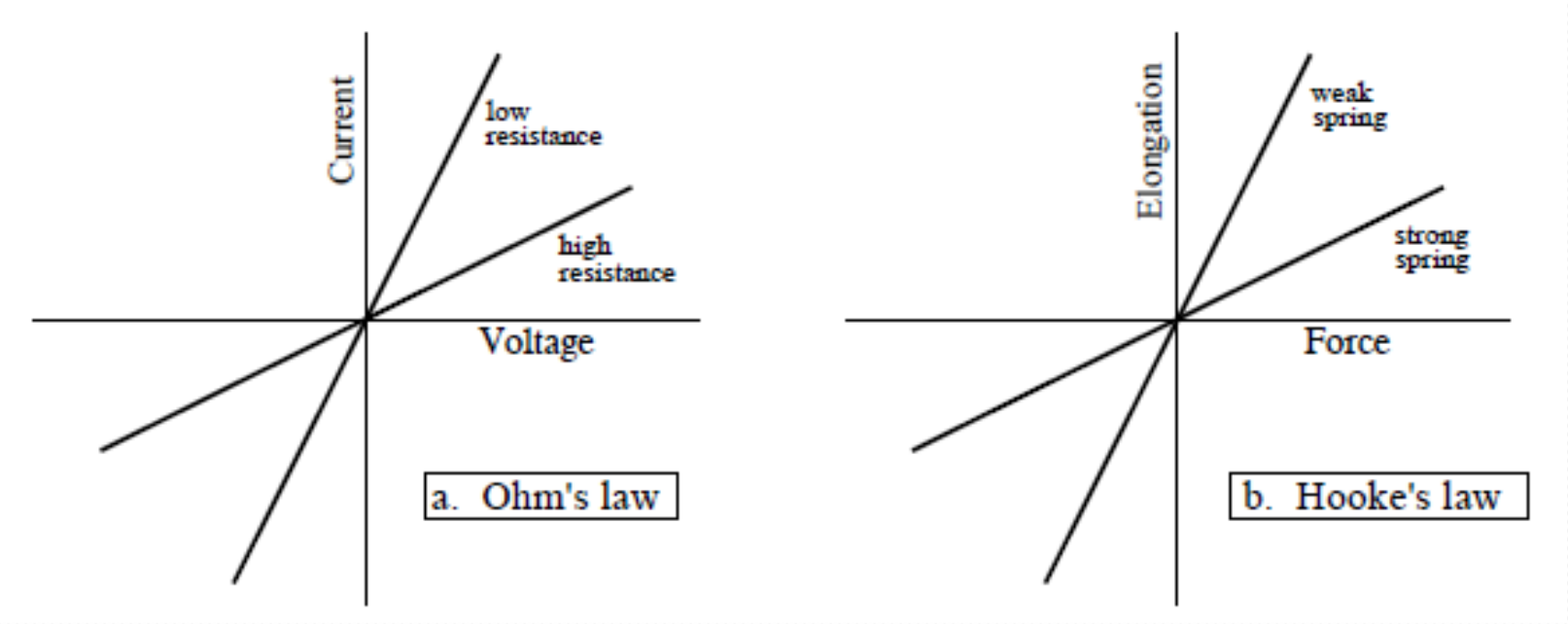
THEN



Static Linearity

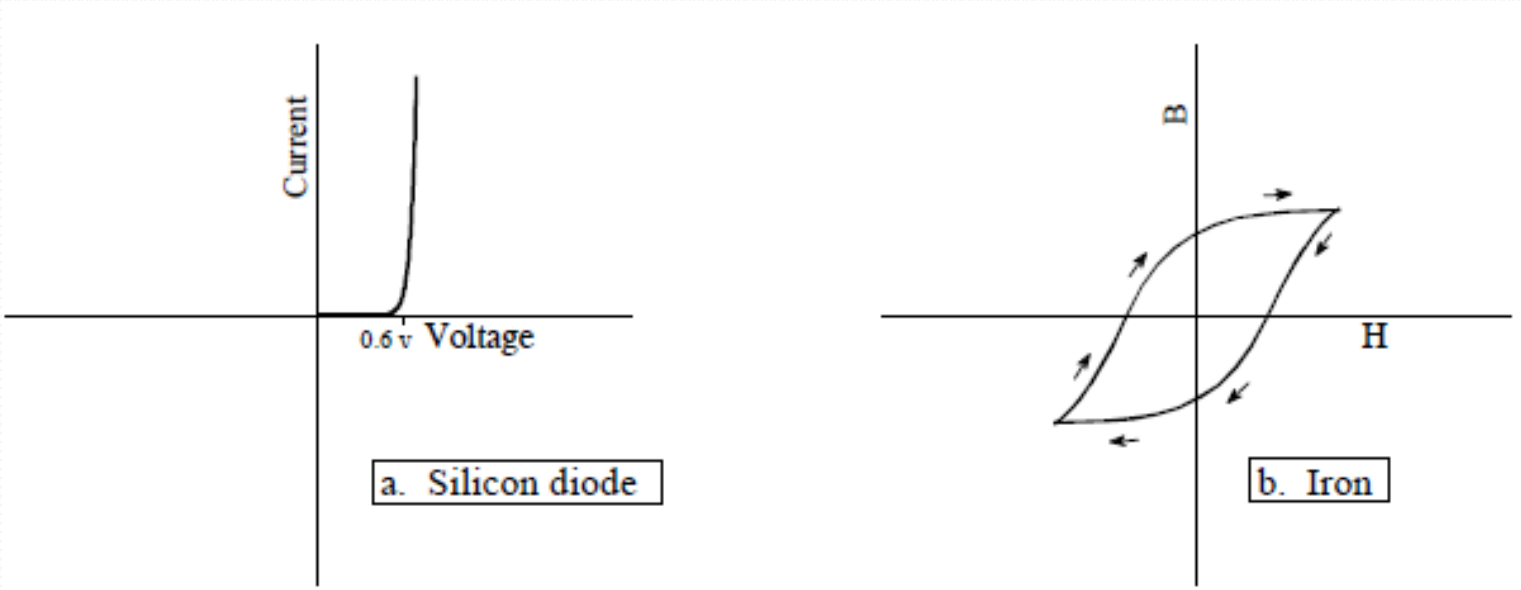
- How the system responses to nonvarying input (DC)?
  - If it is linear → Y=*a*X and *a* is a constant

- Linear System → Static Linearity but Static Linearity ↛ Linear System



Memoryless systems

- The output depends only on instantaneous input not the history



How to prove Linearity (until now)

- Homogeneous + Additive = Linear
- Static Linearity + Memoryless → Linear
- Linear → Static Linearity

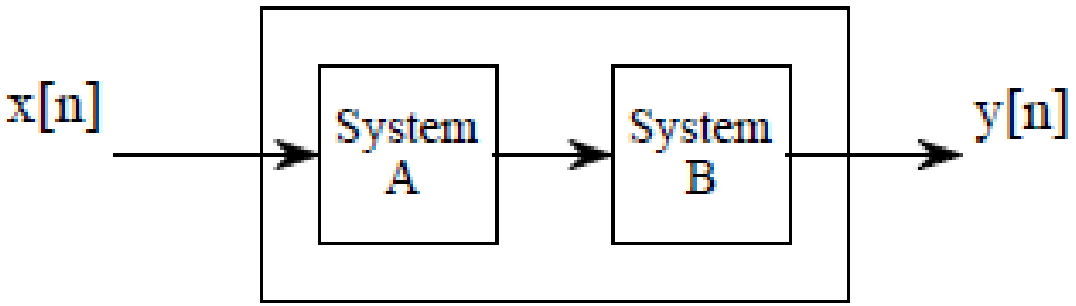
Sinusoidal Fidelity

- Linear system → sinusoidal output for sinusoidal input
- Sinusoidal Fidelity → Linear System
  - (e.g. phase Lock Loop)

- This is why we can work with AC circuits using only two numbers (amplitude and phase)
- This is why Fourier Analysis is important
- This is partially why Linear Systems are important
- This is why you cannot see DSP without *sin*

Properties of Linearity- Commutative

IF



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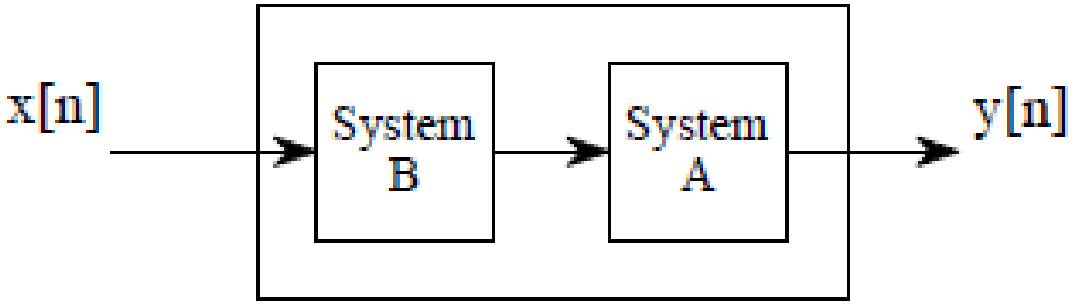
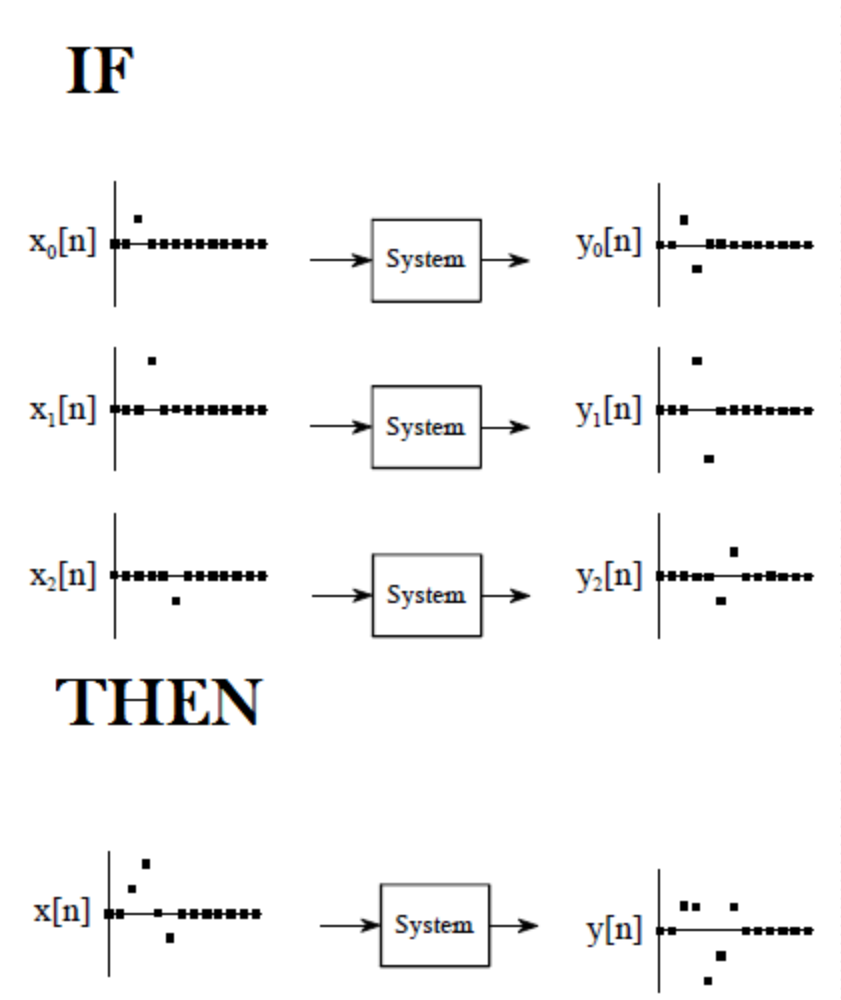
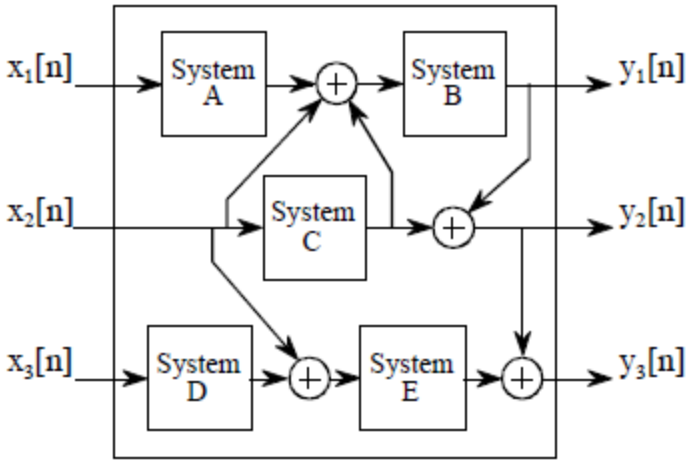


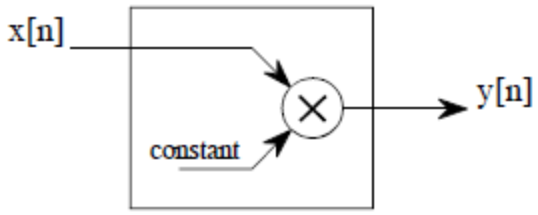
Properties of Linearity – Superposition



Properties of Linearity – Multiple inputs and/or outputs

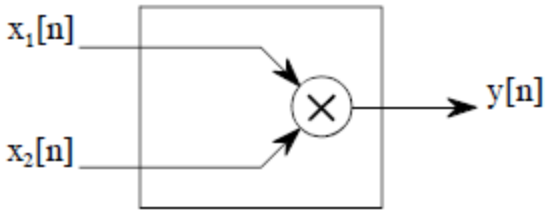
- Linear iff it can be decomposed into linear subsystems connected with only additions





Linear

a. Multiplication by a constant



Nonlinear

b. Multiplication of two signals

Synthesis and Decomposition

- Synthesis
  - Combine signals to produce complex ones
- Decomposition
  - Decompose complex signals into simpler ones

Fundamental Concept of DSP

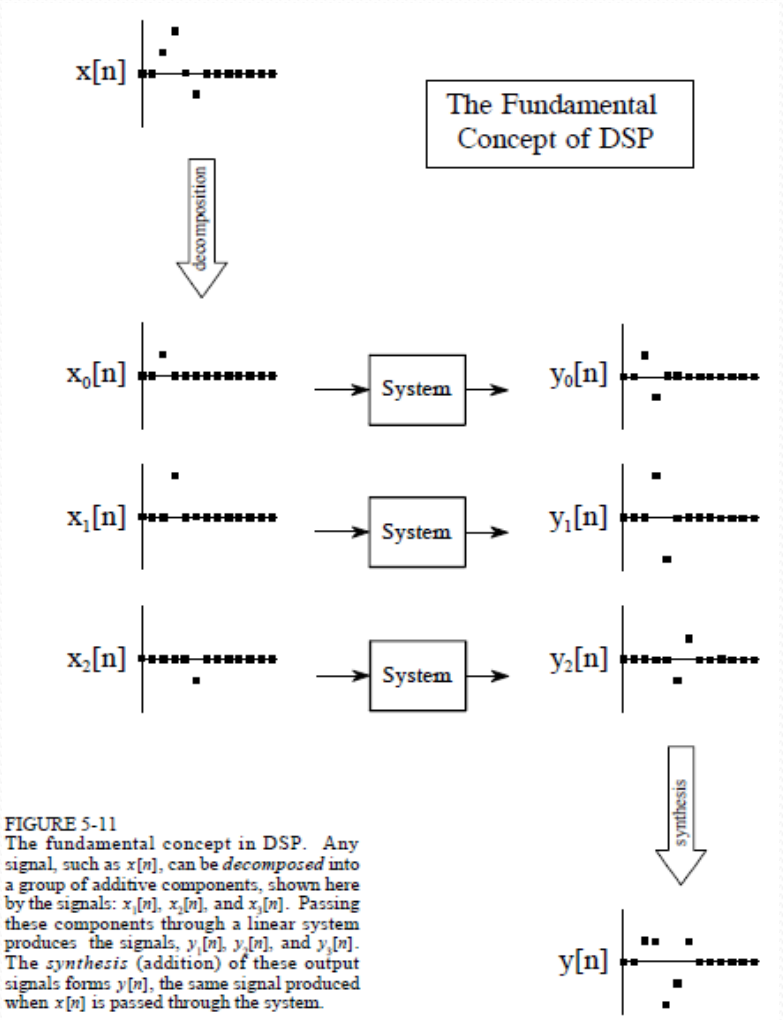
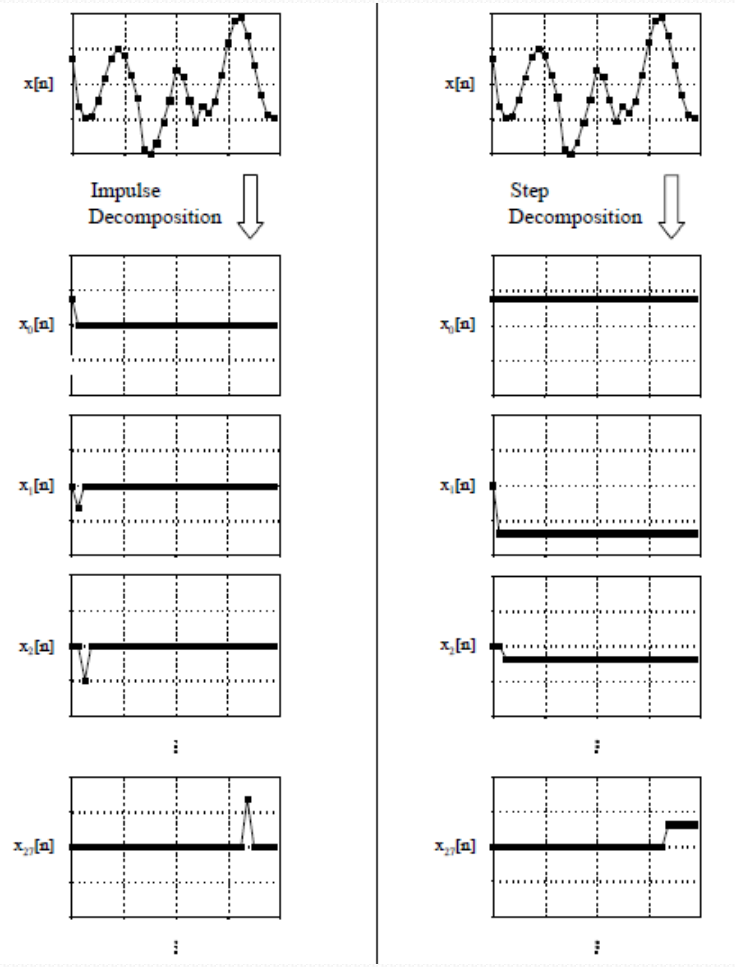


FIGURE 5-11
The fundamental concept in DSP. Any signal, such as $x[n]$, can be *decomposed* into a group of additive components, shown here by the signals: $x_0[n]$, $x_1[n]$, and $x_2[n]$. Passing these components through a linear system produces the signals, $y_0[n]$, $y_1[n]$, and $y_2[n]$. The *synthesis* (addition) of these output signals forms $y[n]$, the same signal produced when $x[n]$ is passed through the system.

Common Decompositions

1. Impulse Decomposition
2. Step Decomposition
3. Even/Odd Decomposition
4. Interlaced Decomposition
5. Fourier Decomposision

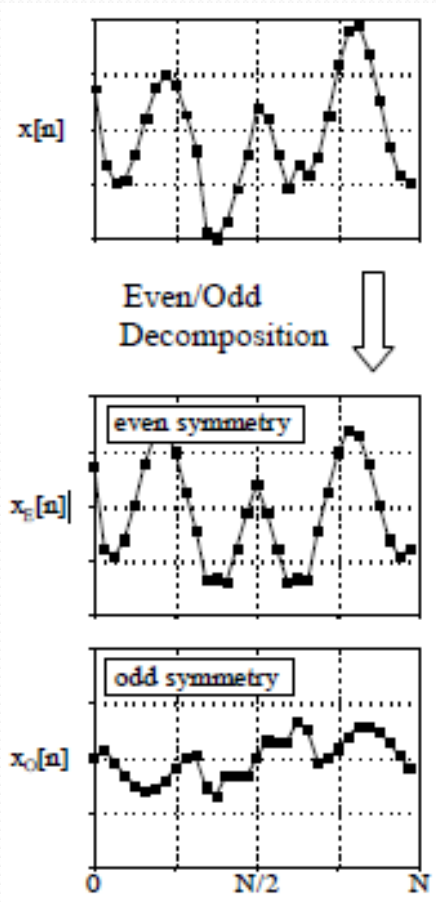
Impulse and Step Decompositions

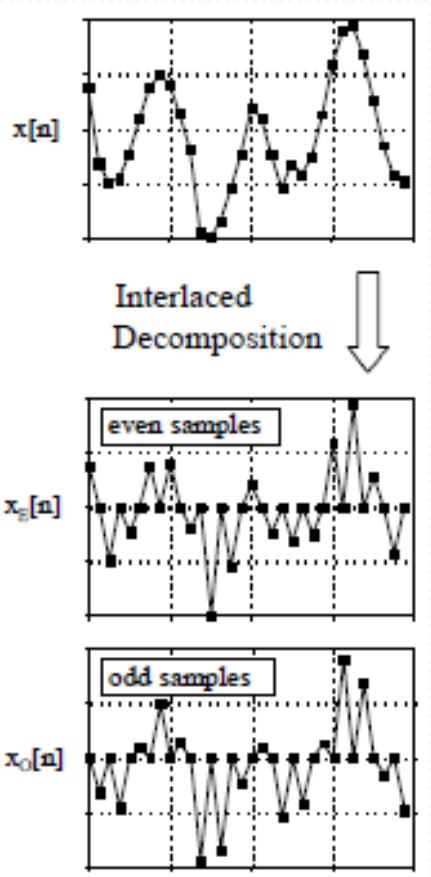


Even/Odd and Interlaced

$$x_E[n] = \frac{x[n] + x[N-n]}{2}$$

$$x_O[n] = \frac{x[n] - x[N-n]}{2}$$





FFT

Fourier Decomposition

- Why sinusoidal?

- Periodic Time Domain → Discrete Frequency Domain
- Discrete Time Domain → Periodic Frequency Domain

		Periodicity	
Continuity		Periodic	aperiodic
	continuous	Fourier Series Aperiodic Spectrum Discrete Spectrum	Fourier Transform Aperiodic Spectrum Continuous Spectrum
	discrete	Discrete Fourier Transform Periodic Spectrum Discrete Spectrum	Discrete Fourier Transform Periodic Spectrum Continuous Spectrum

What if it was not linear?

- First (and usually last) option
  - Assume it is linear
    - If nonlinearity is small it will work (some times even if it is large!!!!)
  - Keep it small
  - Keep it short
  - Linearize it
    - E.g. take the log to convert * into +